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New Gear-Fault-Detection Parameter by Use of Joint Time-Frequency Distribution

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A new gear-fault-detection parameter called NP4 is introduced. This fault-detection parameter utilizes the properties of the joint time–frequency analysis given by the Wigner–Ville distribution (WVD) and kurtosis. With the WVD, the instantaneous power of the gear-vibration signature for one complete rotor revolution can be obtained. The presence of single-gear-tooth damage can be manifested by the existence of an instantaneous power distribution with a peakedness larger than the normal distribution. The normalized kurtosis, a fourth-order statistical parameter calculated for the instantaneous power distribution, provides the gear-fault-detection parameter NP4. The developed fault-detection parameter NP4 is sensitive to gear-tooth damage, especially for damage in a single tooth. The application of this NP4 fault-detection parameter was demonstrated by experimental data obtained from a gear test rig. The results showed that the NP4 parameter, used with the WVD, can provide an accurate fault identification of gear-tooth damage. The parameter NP4 would be of help to the practitioners in the field of machine health monitoring.

Nomenclature

C	= positive real constant
E	= total energy of the signal
f	= frequency in hertz
$h(t)$	= window function
j	= complex number
L	= length of the data window
N	= number of data points
NK	= normalized kurtosis
NP4	= normalized kurtosis of instantaneous signal power
n, i	= index corresponding to the data point
$P(t)$	= instantaneous signal energy or signal power
$\bar{P}$	= mean value of signal power $P(t)$
$s(t)$	= analytic signal, complex function
T	= time sampling interval
t	= time in seconds
$W_{xx}(t, f)$	= Wigner–Ville distribution, a function of both time and frequency
$X(f)$	= energy density spectrum
$x^*(t)$	= complex conjugate of the analytic signal $x(t)$
$x(t), y(t)$	= acquired vibration signal
$\mu(t)$	= weighting function
σ	= standard deviation
σ_t	= positive real constant
τ	= time in seconds

I. Introduction

CURRENTLY many digital signal processing techniques are being developed and applied to gear-fault detection and machine-health diagnosis. Nevertheless, with the advances in signal processing, more accurate fault-detection techniques are being developed every day. In the area of gear damage identification, the researchers are continually looking for better techniques for fault detection and classification. Some of the newly developed procedures are refined versions of previously developed techniques, whereas other methods are applications of the new signal processing algorithms. In this paper, we present the gear-fault-detection parameter, which is based on a combination of several previously developed methodologies.

The signal processing methods for machine-health monitoring can be classified into the time-domain analysis, the frequency-domain analysis and the joint time–frequency-domain analysis. One commonly used group of time-domain techniques for early detection of gear-tooth damage is called figures of merit. Examples of figures of merits are FM0, FM4, NA4, NA4*, NB4, and NB4* parameters.[1–4] The FM0 parameter[1,2] is based on a comparison of the maximum peak-to-peak amplitude with the sum of the gear-mesh frequencies and its harmonics. FM4 is an isolated fault-detection parameter[1,2] given by normalized kurtosis. NA4 is a general fault-detection parameter[3,4] with trending capabilities. In the NA4* method, the trending capabilities of NA4 are enhanced by comparison of the kurtosis of the current signal with the variance of the locked baseline signal under nominal conditions. NB4 is another parameter[3,4] similar to NA4 that also uses the quasi-normalized kurtosis. Again, as with NA4*, NB4* is an enhancement to the NB4 parameter, in which the value of the average variance is locked when the instantaneous variance exceeds a predetermined value.

The Wigner–Ville distribution (WVD) is an example of the joint time–frequency signal representation.[5] Some success has been achieved by the researchers who used the WVD of vibration signals to detect gear-tooth faults.[6–8] The joint time–frequency distribution of the faulty gear shows the vibration energy dispersion at times when damaged gear teeth are in mesh. Such energy changes in time can be detected by conventional fast Fourier transform (FFT) power spectrum analysis as the appearance of sidebands near the gear-mesh frequency at only the advanced stage of gear-tooth damage in the system. The WVD can provide a general indication of gear-tooth failure and its location at a much earlier stage than FFT analysis. However, interpretation of the WVD is not a trivial task. This paper introduces the parameter called NP4, which can simplify the analysis of the WVD for gear-tooth fault detection. The developed NP4 parameter utilizes the advantages given by both the joint time–frequency distribution and the construction of the figures of merits.

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II. Signal Presentation with Joint Time–Frequency Wigner–Ville Distribution

A. Introduction to Computation of the Wigner–Ville Distribution

The WVD is one of the most general time–frequency analysis techniques, as it provides excellent resolution for accurate examination in both time and frequency domains. The WVD produces instantaneous frequency components with rotor rotation, whereas the traditional FFT provides an average spectrum. The extensive studies in signal processing have been made to address the problems associated with the computation of the WVD. To avoid the aliasing problem arising in the computation of the WVD, the original real signal is transformed into the complex analytic signal.[9] Another major obstacle in the application of the WVD is due to its nonlinear behavior. The nonlinearity of the WVD causes the interference between different frequency components of the acquired signal. As a result, the appearance of the interference terms in the WVD can further complicate its interpretation.

The WVD can be written as[9]

$$W_{xx}(t, f) = \int_{-\infty}^{\infty} x\left(t + \frac{\tau}{2}\right)x^*\left(t - \frac{\tau}{2}\right)\exp(-j2\pi f\tau)\,\mathrm{d}\tau \quad (1)$$

where $W_{xx}(t, f)$ is the WVD of a complex continuous-time analytic signal $x(t)$. It follows from Eq. (1) that at time zero the WVD can be calculated as

$$W_{xx}(0, f) = \int_{-\infty}^{\infty} x\left(\frac{\tau}{2}\right)x^*\left(-\frac{\tau}{2}\right)\exp(-j2\pi f\tau)\,\mathrm{d}\tau \quad (2)$$

Equation (2) is used to calculate the WVD at any time point t simply by shifting the time signal origin to that point. The discrete form of the WVD is given as[9]

$$W_{xx}(nT, f) = 2T\sum_{i=-L}^{L} x(nT + iT)x^*(nT - iT)\exp(-j4\pi f i\tau) \quad (3)$$

When the convention is adopted that the sampling period is normalized to unity, Eq. (3) can be rewritten as[9]

$$W_{xx}(n, f) = 2\sum_{i=-L}^{L} x(n + i)x^*(n - i)\exp(-j4\pi f i) \quad (4)$$

Similar to Eq. (2) for the continuous case, the WVD at time zero is found as

$$W_{xx}(0, f) = 2\sum_{i=-L}^{L} k(i)\exp(-j4\pi f i) \quad (5)$$

where $k(i) = s(i)s^*(-i)$ is called the discrete WVD kernel sequence.[9] To suppress the interference terms in the WVD, a weighting function is added to the calculation of the WVD kernel.[6] In the continuous case, the WVD with the added weighting functions $\mu(\tau)$ is

$$W_{xx}(t, f) = \int_{-\infty}^{\infty} x\left(t + \frac{\tau}{2}\right)x^*\left(t - \frac{\tau}{2}\right)\mu(\tau)\exp(-j2\pi f\tau)\,\mathrm{d}\tau \quad (6)$$

$$\mu(\tau) = h(\tau/2)h^*(-\tau/2) \quad (7)$$

$$h(\tau) = C\exp(-\sigma_t^2\tau^2) \quad (8)$$

Although this process may decrease the resolution of the distribution, it also reduces the repetition of the WVD in the time domain and thus makes the interpretation of the WVD easier.

It can be shown that the WVD is a real function but not necessarily a positive function at each point on the time–frequency domain. It would be more convenient to work with a positive function as in the case of the magnitude of the FFT. The WVD can be artificially made positive by simply calculating its absolute value at each point. Practical experience shows that the use of the absolute values of the WVD function simplifies the analysis and the display of the distribution. It also allows the common interpretation of the WVD as energy density or intensity of a signal simultaneously in time and frequency. From this point on, the absolute value of the WVD function is referred to simply as the WVD.

With the energy density interpretation of the WVD, the signal energy at time t and frequency f contained in a cell $\mathrm{d}t$ by $\mathrm{d}f$ can be found as $|W_{xx}(t, f)|\,\mathrm{d}t\,\mathrm{d}f$. Thus the total energy of the signal can be defined through the WVD by integration of the WVD over the time–frequency plane as

$$E = \iint |W_{xx}(t, f)|\,\mathrm{d}t\,\mathrm{d}f \quad (9)$$

Other important signal characteristics that can be derived from the WVD include the energy density spectrum,

$$X(f) = \int |W_{xx}(t, f)|\,\mathrm{d}t \quad (10)$$

and the instantaneous energy of the signal or signal power,

$$P(t) = \int |W_{xx}(t, f)|\,\mathrm{d}f \quad (11)$$

B. Examples of Wigner–Ville Distribution

From the preceding discussion it follows that the WVD is a real-valued function of time and frequency, i.e., the WVD can be viewed as an image or a matrix. Each value of the WVD image is represented by one of the intensities on a color or gray scale. A logarithmic scale is often used to display the WVD to reveal the details of the signal structure that otherwise could be unnoticed on a linear scale.

The measured vibration/acoustic signals from the rotating machinery are often analyzed over a number of rotations from a key phaser (a trigger) on the rotating shaft. With the location of the key phaser defined as a point of 0 deg, the measured signal over one revolution of rotating machinery is displayed from 0 to 360 deg. A key phaser provides a reference point for the comparison between data sets. To compare the WVD images, of the measured signals from different experiments, the acquired signals are normalized with the maximum amplitude to the range from −1 to 1. The normalization simplifies the WVD analysis when applied to a variety of cases. The normalization allows us to work with relative units and to compare the WVD structures easily, regardless of the physical process selected for measurements (e.g., structural vibrations, machine noise), quantity (e.g., displacement, pressure), measurement units of the signal, or the signal amplitude variations. For example, the comparison of the WVDs of accelerometer and microphone measurements for comparison of the signal structure might be difficult as not only are the units of measurements different, but also the range of measured values. The described normalization procedure eliminates the obstacles for comparison of the signals.

In addition to the normalized signal, the WVD, and the color or gray scale, the instantaneous signal power and FFT of the signal are shown in the developed procedure. The examples presented in Figs. 1 and 2 help to elucidate the WVD structure for two types of signals typically encountered in the gear-fault analysis. Figure 1 shows the WVD of a pure sine wave with no changes in the signal. The WVD, the FFT, and the instantaneous signal power confirmed that there were no changes in the signal structure. Figure 2 illustrates the effect of a small phase change on the time signal and the changes observed in the WVD, FFT, and the instantaneous signal power. In this case the frequency spectrum given by the FFT did not produce any visible changes. The small phase change also did not produce substantial changes in the time signal in Fig. 2. Meanwhile, the WVD showed a characteristic diamondlike shape patterns in Fig. 2. The instantaneous signal power plot in Fig. 2 produced the sharp spike. Small phase changes in the signal might simulate uniform wear of a gear tooth.

III. Fault Detection Procedure with Parameter NP4

A. Description of Fault-Detection Parameter NP4

The examples given in Figs. 1 and 2 demonstrated that the peakedness in the instantaneous signal power obtained from the WVD can

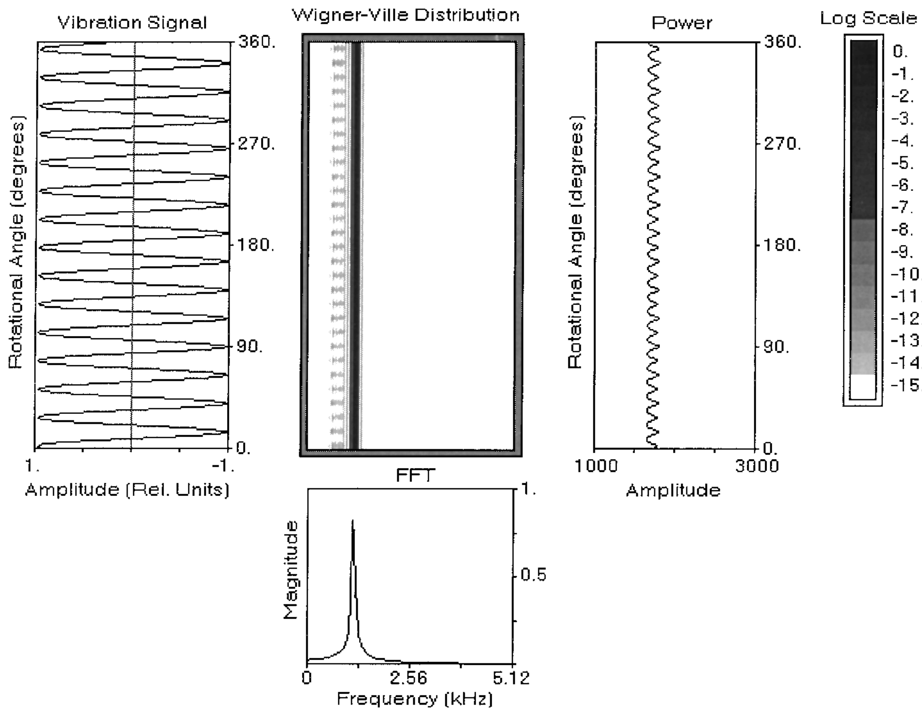

Fig. 1 WVD of a sine-wave time signal.

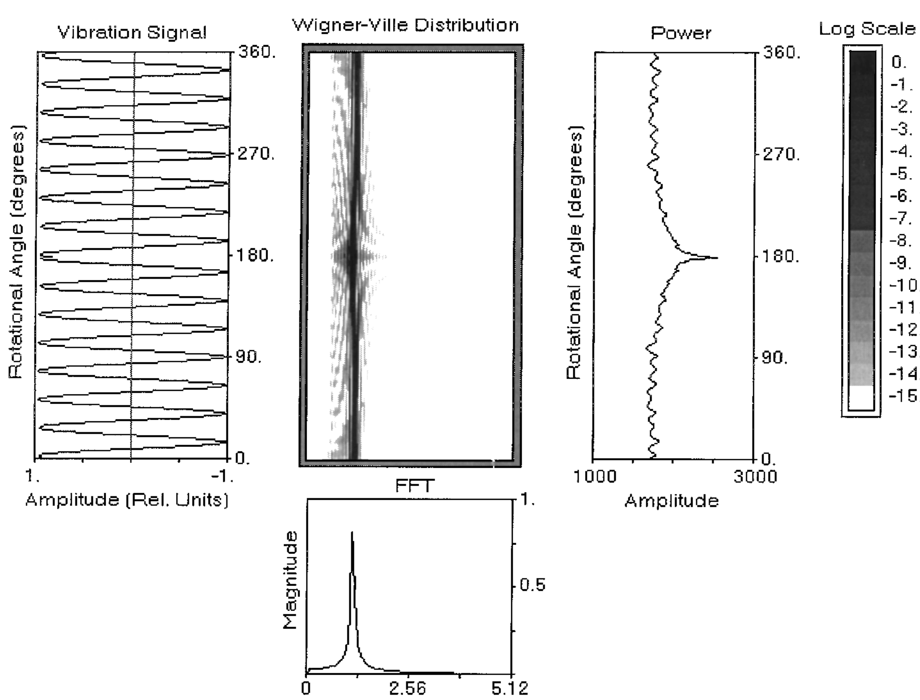

Fig. 2 WVD of a time signal with short-term phase change.

possibly serve as an indication of a single-gear-tooth fault. A statistical parameter, which measures the peakedness or flatness of the distribution, is called kurtosis.[10] Kurtosis is a fourth-order statistical moment. The normalized kurtosis for a distribution $y(t)$ given by its N values $y_1, \ldots, y_N$ measured at times $t_1, \ldots, t_N$ can be defined as

$$\mathrm{NK}[y(t)] = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{y_i - \bar{y}}{\sigma}\right)^4 - 3 \tag{12}$$

where $\bar{y}$ is a mean value, and σ is a standard deviation of $y(t)$ (Ref. 10). Kurtosis as defined by Eq. (12), is zero for a normal distribution and -1.5 for a pure sine wave. Kurtosis is a nondimensional quantity. It is positive for a distribution consisting of a sharp single peak and is increasing with an increase in peakedness of a distribution. This property of the kurtosis was used in the construction of the figures of merits and parameters NA4 and NB4 (Refs. 2–4) to detect sudden sharp changes in the gear-vibration signal.

To utilize the sensitivity of the WVD to signal changes, let us define a parameter NP4 as a normalized kurtosis of signal power $P(t)$ given by Eq. (11) as

$$\mathrm{NP4} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{P(t_i) - \bar{P}}{\sigma}\right)^4 - 3 \tag{13}$$

where σ is a standard deviation of $P(t)$. NP4 is a nondimensional parameter. The parameter NP4 depends on only the shape of the power distribution $P(t)$. Rescaling of the original vibration signal $x(t)$ by multiplying it by a constant does not affect the value of NP4. The same statement is true for scaling of signal power $P(t)$, i.e., multiplication of the signal power by a constant does not change the value of the parameter NP4. In short, the parameter NP4 is invariant to scale transformation. The scale-invariance property of this fault-detection parameter greatly simplifies its usage in practice.

As can be seen from Fig. 1, the instantaneous signal power for a sine-wave time signal is a sine wave with small amplitude. The small fluctuations in the instantaneous signal power were due to the WVD aliasing problems. The NP4 parameter for a sine-wave time signal was -1.5, as referenced in Table 1. In the case of a signal with a small phase change, the sharp peak in the instantaneous power distribution caused the NP4 parameter to be relatively large (compared with a normal distribution) and positive number 7. Thus, the NP4 parameter senses that the overall distribution is sharper than a normal distribution by turning positive.

B. NP4 Fault Detection by Experimental Data

The gear-fault-detection parameter NP4 performance was conducted with the experimental data obtained from a helicopter tail gear transmission driven by two T-700 engines. Two identical sets of experiments were carried out on the gear test rig under various loading conditions. Figures 3–6 show the WVD for the accelerometer data measured for both series of the experiments at the gear load 2000 lb ft; the first is using a perfectly undamaged transmission, and the second is having single-tooth damage in the tail gear, as shown in Fig. 7.

Table 1 NP4 values for data shown in Figs. 1–6

Figure number	NP4 value
1	-1.5
2	7.0
3	0.0
4	-1.0
5	4.5
6	3.3

To increase the reliability of the gear-fault detection with NP4 parameter, its detection capabilities could be enhanced in a similar manner as for the FM4 and NA4 parameters.[4] Most of the rotating machinery, including gear boxes, produces complex vibration signals with many components. Some of these vibration signal components might strongly indicate faulty gear conditions if they are viewed on an appropriate energy scale without being attenuated by the signal components with a higher energy content. A residual vibration signal used to construct FM4 and NA4 parameters can help to investigate the vibration signal energy distribution. In this study, a residual vibration signal of the order of N was constructed by the filtering out of the N largest signal FFT components in a frequency domain. For example, to construct the first-order residual signal, the FFT is calculated. Then the largest FFT component is removed from the spectrum and the inverse FFT is applied, giving a first-order residual signal. The NP4 parameter calculated for the Nth-order residual signal is denoted as NP4(N). The NP4 parameter for an original vibration signal is referred to as NP4(0). Usually the largest FFT component for a gear-vibration signal appears at the first gear-mesh frequency.

Let us introduce a possible fault detection strategy with the NP4 parameter for single-gear-tooth damage:

1) Calculate NP4(0).

2) If NP4(0) is greater than zero and large, then set a fault alarm.

3) If NP4(0) is smaller than zero, but larger than -1, a multiple gear-tooth-fault condition possible.

4) Calculate NP4(1) for confirmation of NP4(0).

5) If NP4(1) is greater than zero and large, then set a fault alarm. Otherwise the result of NP4(0) is not confirmed.

This strategy can be refined by calculation of the residual signal of higher order, setting the thresholds other than 0 and -1, etc. The primary goal of the given strategy was to detect single-gear-tooth damage. It was expected that NP4 can provide a warning of the gear-tooth-damage initiation.

Figures 3 and 4 show the WVD and the instantaneous power plot for a vibration signal of an undamaged gear. The FFT shows two large-frequency components, which in this case are the first and the second harmonics of the gear-mesh frequency. Because the

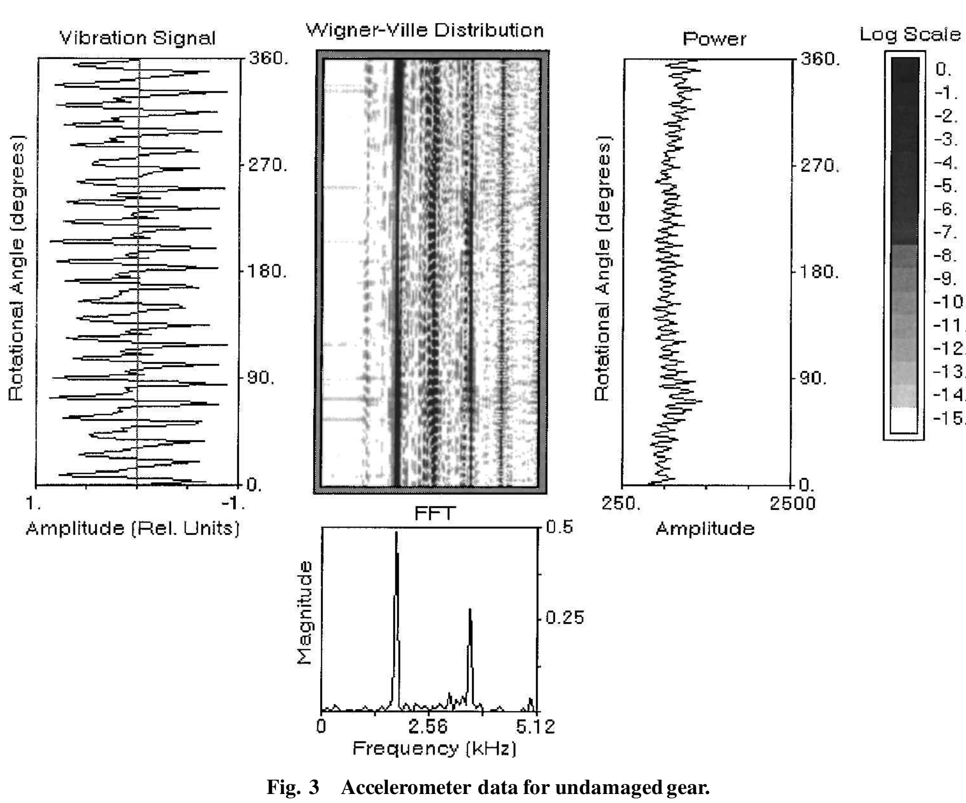

Fig. 3 Accelerometer data for undamaged gear.

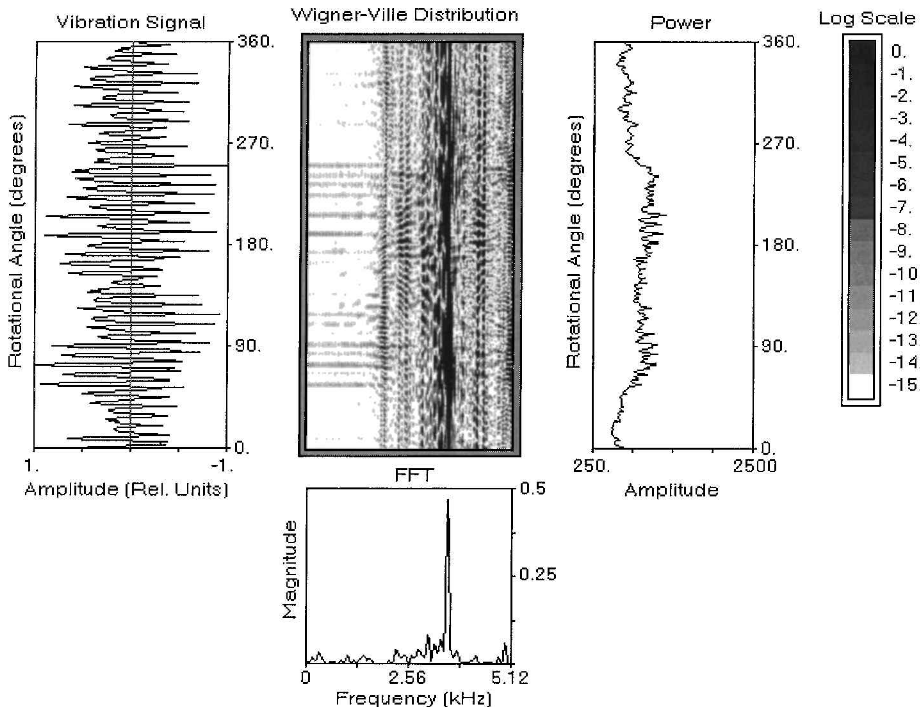

Fig. 4 Accelerometer data for undamaged gear with filtered-out gear-mesh frequency.

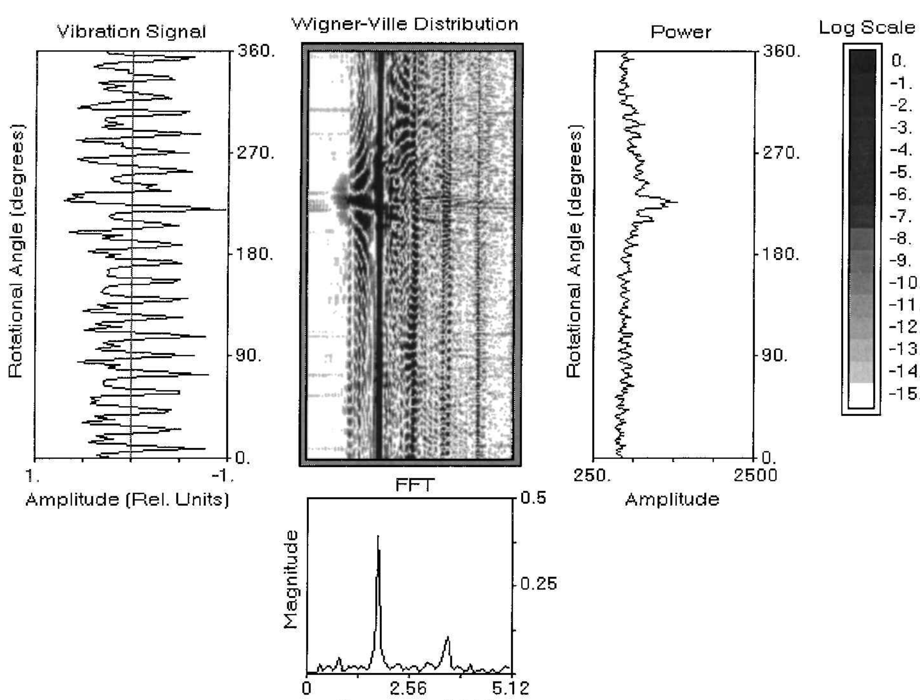

Fig. 5 Accelerometer data for damaged gear.

NP4(0) parameter was zero in this case, the parameter NP4(1) was calculated. The NP4(1) = −1.0 confirmed that the gear does not have a single-tooth damage. The dashed line between the gear-mesh frequencies in Fig. 3 was produced by the interference terms in the WVD.

The WVD and the instantaneous power plot for a damaged gear are shown in Figs. 5 and 6. The parameter NP4(0) was equal to 4.5, which flagged a fault alarm. The parameter NP4(1) = 3.3 confirmed the presence of single-gear-tooth damage. As shown in Figs. 4 and 6, a faulty gear tooth produced an energy concentration around the gear-mesh frequency at the location of the damage between 180 and 270 deg. This energy dispersion at the first gear-mesh frequency is especially noticeable in Fig. 6. An irregular power transmitted through a damaged gear caused the excitation of the gearbox frequencies around 500 Hz other than the first gear mesh for a short duration. As the undamaged gear teeth went into mesh, the excitation around 500 Hz and between 180 and 270 deg decayed. Figure 7 documents the gear-tooth damage, which produced the damage signatures shown in Figs. 5 and 6. A summary of NP4 values for the signals presented in Figs. 1–6 is given in Table 1.

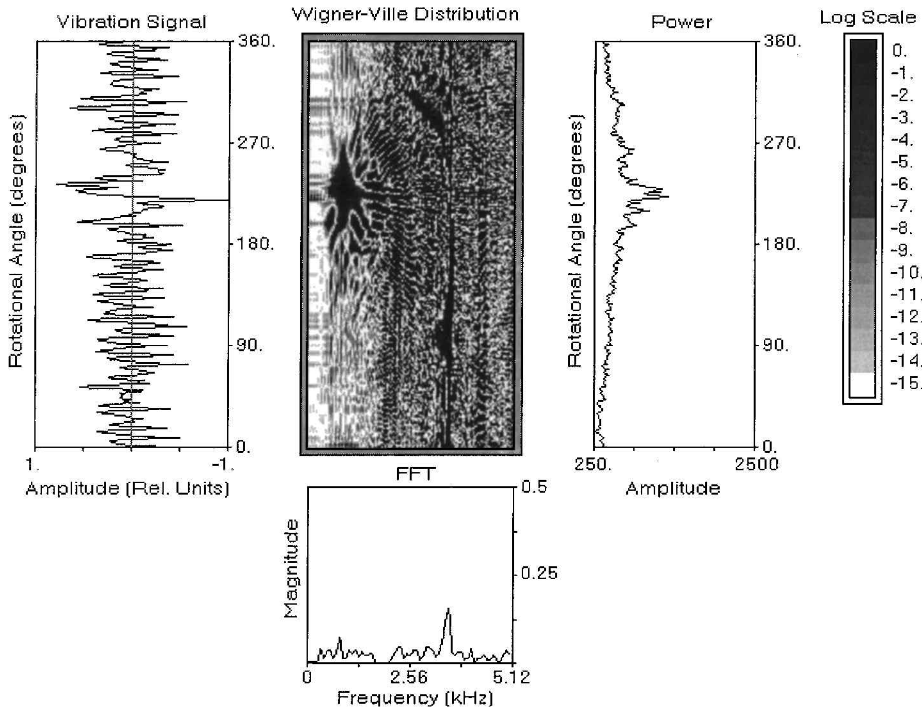

Fig. 6 Accelerometer data for damaged gear with filtered-out gear-mesh frequency.

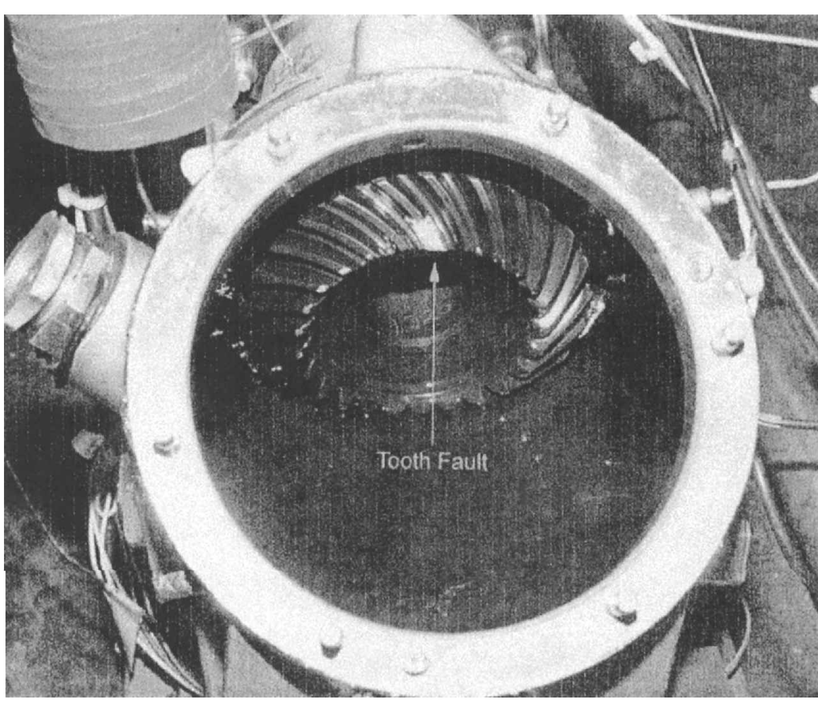

Fig. 7 Photograph of the damaged gear.

IV. Conclusions

This paper introduced a new gear-fault-detection parameter and demonstrated its application on the data obtained from an experimental test rig. It has been shown that the gear-fault-detection parameter NP4 is sensitive to single-gear-tooth damage. The newly introduced parameter NP4 could simplify interpretation of the Wigner–Ville joint time–frequency distribution. Vibration analysis specialists can use the parameter NP4 to detect the gearbox faults or abnormalities in the performance of the rotating machinery.

The relationship between the NP4 parameter and the severity of the gear damage as well as other operating conditions of the gearbox require further study. Such studies could quantify the parameter NP4 with respect to the amount and the type of the gear-tooth damage.

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